Exponential distribution:

A continuous random variable x is said to follow exponential distribution if its probability density function is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & \lambda > 0,\ x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Distribution function (F(x)):

$$F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{0}^{x} \lambda e^{-\lambda x}\,dx = \lambda\left[\frac{e^{-\lambda x}}{-\lambda}\right]_{0}^{x}$$

$$= -e^{-\lambda x} + e^{0}$$

$$F(x) = 1 - e^{-\lambda x}$$

Moment Generating Function (MGF):

$$M_X(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} f(x)\,dx$$

$$= \int_{0}^{\infty} e^{tx} \lambda e^{-\lambda x}\,dx$$

$$= \lambda \int_{0}^{\infty} e^{-(\lambda - t)x}\,dx$$

$$= \lambda\left[\frac{e^{-(\lambda-t)x}}{-(\lambda-t)}\right]_{0}^{\infty}$$

$$= \lambda\left[\frac{e^{-\infty}}{-(\lambda-t)} - \frac{e^{0}}{-(\lambda-t)}\right]$$

$$M_x(t) = \lambda\left[0 + \frac{1}{\lambda - t}\right]$$

$$\boxed{M_x(t) = \frac{\lambda}{\lambda - t}}$$

Mean :

$$E(x) = \left[\frac{d}{dt} M_x(t)\right]_{t=0}$$

$$= \left[\frac{d}{dt}\left(\frac{\lambda}{\lambda - t}\right)\right]_{t=0}$$

$$= \left[\frac{-\lambda}{(\lambda - t)^2}(-1)\right]_{t=0}$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$$

$$E(x) = \frac{\cancel{\lambda}}{\cancel{\lambda^2}}$$

$$\boxed{E(x) = \frac{1}{\lambda}}$$

Variance :

Let $E(x^2) = \left[\frac{d^2}{dt^2} M_x(t)\right]_{t=0} = \frac{d}{dt}\left[\frac{d}{dt} M_x(t)\right]_{t=0}$

$$= \left\{\frac{d}{dt}\left[\frac{\lambda}{(\lambda - t)^2}\right]\right\}_{t=0}$$

$$\frac{d}{dx}\left(\frac{1}{x^2}\right) = \frac{-2}{x^3}$$

$$E(x^2) = \left[\frac{-2\lambda}{(\lambda-t)^3}(-1)\right]_{t=0}$$

$$E(x^2) = \frac{2\lambda}{\lambda^3} = \frac{2}{\lambda^2}$$

$$V(x) = E(x^2) - (E(x))^2$$

$$= \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2$$

$$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2}$$

$$\boxed{V(x) = \frac{1}{\lambda^2}}$$

Normal Distribution :

The probability density function of the normal distribution is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty.$$

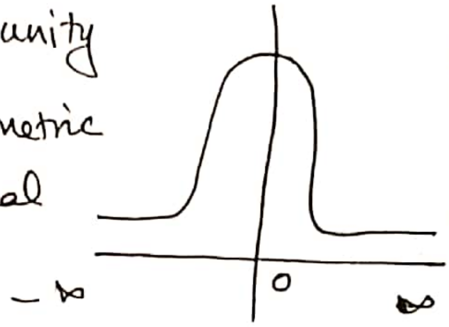
where μ - Mean σ - std.

If x is normally distributed rv with parameters μ and σ then $z = \frac{x-\mu}{\sigma}$ is the standard normal Variate with mean $\mu = 0$ and std deviation $\sigma = 1$

CHARACTERISTICS OF NORMAL DISTRIBUTION:

* Area under normal curve is unity

* Normal distribution is a symmetric distribution and the graph of normal distribution is bell shaped.



* Mean of the normal distribution lies at the centre of the normal curve

* In normal distribution mean, median, mode coincides.

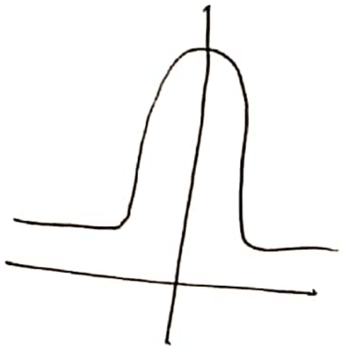
*

Area property:

* $P(-\infty < z < 0) = P(0 < z < \infty) = 0.5$

* $P(-\infty < z < -z_1) = P(z_1 < z < \infty)$

$\cancel{P(z_1 < z}$



* $P(-z_1 < z < z_1) = P[|z| < z_1]$

$= 2P[0 < z < z_1]$

* $P[z > z_1] = 0.5 - P(0 < z < z_1)$

* $P[z < z_1) = P[-\infty < z < z_1] = 0.5 + P(0 < z < z_1)$